

# Set 1 – Marking Scheme

## Paper 1

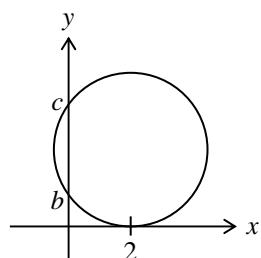
Solution	Marks	Remarks
<b>Section A(1)</b>		
1. $\begin{aligned} \frac{(ab^4)^{-2}}{a^{-3}b} &= \frac{a^{-2}b^{-8}}{a^{-3}b} \\ &= a^{-2-(-3)}b^{-8-1} \\ &= ab^{-9} \\ &= \frac{a}{b^9} \end{aligned}$	2M     1A     <hr style="width: 100px; margin-left: auto; margin-right: 0;"/> 3	For any two of: (i) $(x^m)^n = x^{mn}$ (ii) $\frac{x^m}{x^n} = x^{m-n}$ (iii) $x^{-m} = \frac{1}{x^m}$  Deduct 1A for not expressing answer in positive indices
2. $\begin{aligned} f(0) &= f(k) \\ 0^2 - 4(0) + 3 &= k^2 - 4k + 3 \\ k^2 - 4k &= 0 \\ k(k - 4) &= 0 \\ k = 0 \quad \text{or} \quad 4 & \end{aligned}$	1A     1M 1A     <hr style="width: 100px; margin-left: auto; margin-right: 0;"/> 3	For any appropriate method of solving quadratic equation
3. (a) $\begin{aligned} x^4 + 4x^2 + 4 & \\ &= (x^2 + 2)^2 \end{aligned}$ (b) $\begin{aligned} x^4 + 4 & \\ &= x^4 + 4x^2 + 4 - 4x^2 \\ &= (x^2 + 2)^2 - (2x)^2 \\ &= (x^2 + 2x + 2)(x^2 - 2x + 2) \end{aligned}$	1A     1M 1A     <hr style="width: 100px; margin-left: auto; margin-right: 0;"/> 3	For using (a)
4. (a) $\begin{aligned} C &= \frac{5}{9}(F - 32) \\ \frac{9}{5}C &= F - 32 \\ F &= \frac{9}{5}C + 32 \end{aligned}$ (b) When $C = 40$ , $F = \frac{9}{5}(40) + 32$ $= 104$	1A     1A     1M 1A     <hr style="width: 100px; margin-left: auto; margin-right: 0;"/> 4	

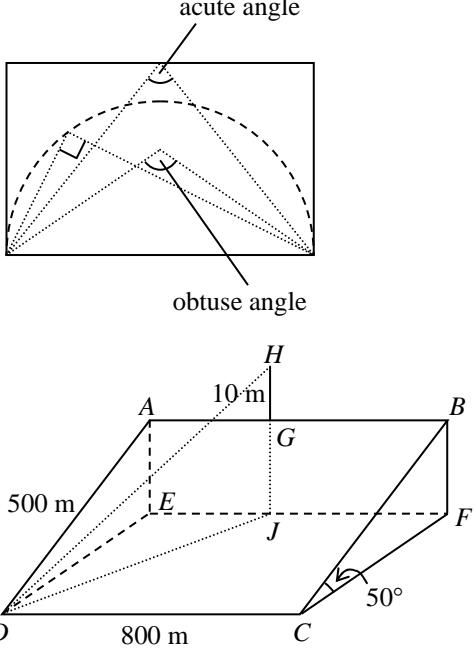
Solution	Marks	Remarks
<p>5.      Area of sector = <math>\pi(15)^2 \left( \frac{360^\circ - 144^\circ}{360^\circ} \right)</math>  <math>= 135\pi \text{ cm}^2</math>  <math>\pi(\text{base radius})(15) = 135\pi</math>  <math>\text{base radius} = 9 \text{ cm}</math>  <math>\text{Height of cone} = \sqrt{15^2 - 9^2}</math>  <math>= 12 \text{ cm}</math>  <math>\text{Volume of cone} = \frac{1}{3}\pi(9^2)(12)</math>  <math>= 324\pi \text{ cm}^3</math></p>	1M 1A 1A 1A <hr/> 4	For $\frac{360^\circ - 144^\circ}{360^\circ}$ } u - 1 for missing unit
<p>6. (a) Let the marked price be \$x.  <math>x(10\%) = 12</math>  <math>x = 120</math>  ∴ The marked price is \$120.</p> <p>(b) Cost = <math>120 \times (1 - 10\%) \div (1 + 35\%)</math>  = \$80</p>	1M 1A 1M 1A <hr/> 4	u - 1 for missing unit OR $(120 - 12) \div (1 + 35\%)$ u - 1 for missing unit
<p>7. (a) Let <math>\angle EBC = b</math> and <math>\angle ECD = c</math>  Then <math>\angle ABC = 2b</math> and <math>\angle ACD = 2c</math>  <math>\angle BEC = 30^\circ = c - b</math> (ext. ∠ of Δ)  <math>\angle BAC = 2c - 2b</math> (ext. ∠ of Δ)  = <math>2(c - b)</math>  = <math>60^\circ</math></p> <p>(b) If <math>AB = AC</math>, <math>\angle ABC = \angle ACB</math> (base ∠s, isos. Δ)  <math>= \frac{180^\circ - 60^\circ}{2}</math> (∠ sum of Δ)  = <math>60^\circ</math>  ∴ ΔABC is equilateral.</p>	1A 1A 1M 1A <hr/> 4	
<p>8. (a) The coordinates of <math>C = \left( \frac{2+(-4)}{2}, \frac{3+1}{2} \right) = (-1, 2)</math></p> <p>(b) Image of <math>C</math> after translation = <math>(-1, 0)</math>  Image after reflection (<math>D</math>) = <math>(-1, -2)</math>  Slope of <math>OA = \frac{3-0}{2-0} = \frac{3}{2}</math>  Slope of <math>BD = \frac{1-(-2)}{-4-(-1)} = -1</math>  ∴ <math>OA</math> is not parallel to <math>BD</math>.</p>	1A 1A 1M 1A <hr/> 4	For finding the slope of the lines

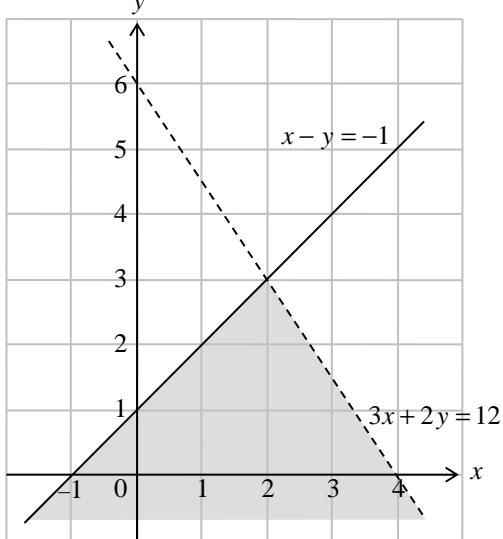
	Solution	Marks	Remarks
9. (a)	$\begin{aligned} & 31+33+40+(40+x)+49+52 \\ & +52+53+55+56+64+65+67 \\ & \frac{+68+68+69+70+72+73+81}{20}=58 \\ & \quad x=2 \\ & \frac{(60+y)+66}{2}=65 \\ & \quad y=4 \end{aligned}$	1M 1A 1M 1A 1A 1A	
(b)	<p><math>v_B</math> is smaller. The scores of 1B students are more concentrated around the mean of the data range.</p>		or any reasonable explanation
		6	
<b>Section A(2)</b>			
10. (a)	<p>Let <math>x = kyz^2</math>, where <math>k</math> is a non-zero constant.</p> $18 = k(4)(3)^2$ $k = \frac{1}{2}$ $\therefore x = \frac{1}{2}yz^2$	1M 1A 1A	pp - 1 for writing $x \propto kyz^2$
(b)	<p>The new value of <math>y = y(1 + 20\%) = 1.2y</math> The new value of <math>z = z(1 + 10\%) = 1.1z</math> New value of <math>x = k(1.2y)(1.1z)^2</math>  <math>= 1.452kyz^2</math>  <math>= 1.452x</math></p> <p>Percentage change in <math>x = \frac{1.452x - x}{x} \times 100\%</math>  <math>= 45.2\%</math></p>	1A 1M 1A	
		6	
11. (a)	<p>Proof:</p> $\begin{aligned} \angle ABD &= 90^\circ \\ \angle BDC &= 180^\circ - 90^\circ \text{ (int. } \angle s, AB \parallel CD) \\ &= 90^\circ \\ \angle BCA &= 90^\circ \quad (\angle \text{ in semi-circle}) \\ \therefore \angle BDC &= \angle ACB \\ \angle ABC &= \angle BCD \quad (\text{alt. } \angle s, AB \parallel CD) \\ \therefore \Delta ABC &\sim \Delta BCD \quad (\text{AAA}) \end{aligned}$	1M 1M 1M 1M	
(b)	<p>Since <math>\Delta ABC \sim \Delta BCD</math>, <math>\frac{AB}{BC} = \frac{BC}{CD}</math>. (corr. sides, <math>\sim \Delta</math>s)</p> $\therefore 8(2) = BC^2$ $BC = 4$ $AC = \sqrt{8^2 - 4^2} = 4\sqrt{3}$ $\therefore \text{Area of } \Delta ABC = \frac{1}{2}(4)(4\sqrt{3}) = 8\sqrt{3}$ $= 13.9 \text{ sq. units}$	1M 1A 1A	
		6	

Solution	Marks	Remarks
<p>12. (a) <math>\Delta = (-4)^2 - 4(1)(-7)</math>  <math>= 44 &gt; 0</math>  <math>\therefore</math> The equation has two distinct real roots.</p> <p>(b) (i) The graph opens upwards as the coefficient of <math>x^2</math> in <math>f(x)</math> is positive.  (ii) <math>f(x) = x^2 - 4x - 7</math>  <math>= x^2 - 4x + 4 - 11</math>  <math>= (x - 2)^2 - 11</math>  <math>\therefore</math> vertex = (2, -11)</p> <p>(c) <math>2x^2 - 3x + 1 = 0</math>  <math>x^2 - \frac{3}{2}x + \frac{1}{2} = 0</math>  <math>x^2 - 4x - 7 = \frac{3}{2}x - \frac{1}{2} - 4x - 7</math>  <math>= -\frac{5}{2}x - \frac{15}{2}</math>  <math>\therefore</math> We should add the straight line <math>y = -\frac{5}{2}x - \frac{15}{2}</math>.</p>	1A 1A 1A 1A 1M	For attempt to transform one side to $x^2 - 4x - 7$
	7	
<p>13. (a) (i) Hemi-spherical surface area <math>= \frac{1}{2} \times 4\pi(6)^2</math>  <math>= 72\pi \text{ cm}^2</math>  (ii) Slant height of conical part <math>= \sqrt{6^2 + 8^2}</math>  <math>= 10 \text{ cm}</math>  Conical surface area <math>= \pi(6)(10)</math>  <math>= 60\pi \text{ cm}^2</math></p> <p>(b) Total cost <math>= 72\pi \times 1 + 60\pi \times 1.5</math>  <math>= \\$509</math> (nearest dollar)</p>	1M 1A 1A 1M 1A 1M 1A	
	7	
<p>14. (a) (i) <math>AD = c \sin B</math>  (ii) Similarly, <math>AD = b \sin C</math>  <math>\therefore b \sin C = c \sin B \Rightarrow \frac{b}{\sin B} = \frac{c}{\sin C}</math>  By symmetry, <math>\frac{a}{\sin A} = \frac{b}{\sin B}</math>.  <math>\therefore \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}</math></p> <p>(b) (i) <math>BD = c \cos B</math>  (ii) By Pyth. thm, <math>AD^2 = AB^2 - BD^2</math>  and <math>AD^2 = AC^2 - DC^2</math></p>	1A 1A 1M 1A	
	1A	

Solution	Marks	Remarks
$\begin{aligned}\therefore AB^2 - BD^2 &= AC^2 - DC^2 \\ &= AC^2 - (BC - BD)^2 \\ c^2 - (c \cos B)^2 &= b^2 - (a - c \cos B)^2 \\ c^2 - c^2 \cos^2 B &= b^2 - a^2 + 2ac \cos B - c^2 \cos^2 B \\ b^2 &= a^2 + c^2 - 2ac \cos B\end{aligned}$	1M 1A	
(c) From (a)(ii), Let $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$ , where $k$ is a non-zero constant. $\therefore a = k \sin A, b = k \sin B, c = k \sin C$ From (b)(ii), $\begin{aligned}b^2 &= a^2 + c^2 - 2ac \cos B \\ k^2 \sin^2 B &= k^2 \sin^2 A + k^2 \sin^2 C - 2k^2 \sin A \sin C \cos B \\ \sin^2 B &= \sin^2 A + \sin^2 C - 2 \sin A \sin C \cos B \\ &= (\sin A + \sin C)^2 - 2 \sin A \sin C \\ &\quad - 2 \sin A \sin C \cos B \\ &= (\sin A + \sin C)^2 - \\ &\quad - 2 \sin A \sin C (1 + \cos B) \\ &< (\sin A + \sin C)^2 \\ \sin B &< \sin A + \sin C \\ \sin(A + C) &< \sin A + \sin C \\ \sin A + \sin C &> \sin(A + C)\end{aligned}$	1M 1M 1M 1A	
	9	
<b>Section B</b>		
15. (a) Since the $x$ -axis is tangent to $\Gamma$ , $EA$ is perpendicular to the $x$ -axis. (tangent $\perp$ radius) $\therefore$ The $x$ -coordinate of $E$ is 2. On the other hand, $E$ lies on the perpendicular bisector of $BC$ . The equation of the perpendicular bisector of $BC$ is: $\begin{aligned}y &= (b + c) / 2 \\ &= 6 / 2 \quad (b \text{ and } c \text{ are the roots of the equation } x^2 - 6x + k = 0) \\ &= 3\end{aligned}$ $\therefore$ The $y$ -coordinate of $E$ is 3. $\therefore E = (2, 3)$	1M 1A 1A 1A	Can be awarded for similar reasoning for the $y$ -coordinate
(b) $\Gamma$ has centre $(2, 3)$ and radius 3. The equation of $\Gamma$ is $(x - 2)^2 + (y - 3)^2 = 3^2$ $x^2 + y^2 - 4x - 6y + 4 = 0$	1A 1A	For radius
	5	



Solution	Marks	Remarks
<p>16. (a) <math>a_4 = 11</math></p> <p>(b) We have <math>a_2 - a_1 = 2</math>, <math>a_3 - a_2 = 3</math>, <math>a_4 - a_3 = 4</math> and so on.  <math>\therefore a_n = a_1 + (a_2 - a_1) + (a_3 - a_2) + \dots + (a_n - a_{n-1})</math>  <math>= 2 + 2 + 3 + \dots + n</math>  <math>= 2 + \frac{n-1}{2}[2(2) + (n-2)(1)]</math>  <math>= \frac{n^2 + n + 2}{2}</math></p>	1A 1A 1M 1M 1A <hr/> 5	(sum of $n - 1$ terms of arithmetic sequence)
<p>17. (a) (i) <math>x = 2</math> or <math>x = 4.25</math>  (ii) No solution  (iii) <math>0 \leq x \leq 1</math> or <math>4.5 \leq x \leq 6</math></p>	1A 1A 2A <hr/> 4	Award 1A if only one interval is given pp -1 if “or” is missing or “and” is used
<p>18. (a) Consider a semi-circle lying on the plane <math>ABCD</math> with <math>DC</math> as diameter. Its radius is 400 m. Since <math>AD = 500</math> m, the circle does not intersect <math>AB</math>. If <math>G</math> is on the circle, then <math>\angle DGC = 90^\circ</math> by angle in semi-circle. Now <math>G</math> lies outside the circle, so <math>\angle DGC</math> must be acute.</p> <p>(b) (i) In <math>\triangle ADE</math>, <math>\sin \angle ADE = \frac{AE}{AD}</math>  <math>\sin 50^\circ = \frac{AE}{500}</math>  <math>AE = 500 \sin 50^\circ</math>  <math>= 383</math> m</p> <p>(ii) Let <math>J</math> be the projection of <math>G</math> to the ground.  <math>EJ = AG = 800 \times \frac{2}{2+3} = 320</math> m  <math>DJ = \sqrt{DE^2 + EJ^2} = \sqrt{(500 \cos 50^\circ)^2 + 320^2}</math>  <math>HJ = HG + GJ = 10 + 500 \sin 50^\circ</math>  <math>\tan \angle HDJ = \frac{HJ}{DJ}</math>  <math>= \frac{10 + 500 \sin 50^\circ}{\sqrt{(500 \cos 50^\circ)^2 + 320^2}}</math>  <math>= 0.866575</math>  <math>\angle HDJ = 40.9^\circ</math>  <math>\therefore</math> The angle of elevation of <math>H</math> from <math>D</math> is <math>40.9^\circ</math>.</p>	1M 1M 1A 1A 1A 1A 1A 1A 1A 1A 1A 1A 1A 1A 1A 1A <hr/> 10	

Solution	Marks	Remarks
19. (a) 		
	1A	For $3x + 2y = 12$ correctly drawn using dotted line
	1A	For $x - y = -1$ correctly drawn using solid line
	1A	For shading the correct region
(b) (i) $(0, 0), (1, 0), (2, 0), (3, 0), (0, 1)$ $(1, 1), (2, 1), (3, 1), (1, 2), (2, 2)$	2A	Award 1 mark for at least 7 correct Deduct 1 mark for more than 1 extra ordered pair
(ii) (1) 6 points do not lie on the $x$ -axis. $P(\text{at least one on } x\text{-axis})$ $= 1 - P(\text{both not on } x\text{-axis})$ $= 1 - \frac{6}{10} \times \frac{5}{9}$ $= \frac{2}{3}$	1M 1M 1A	
<b>OR</b>		
$P(\text{at least one on } x\text{-axis})$ $= P(\text{only 1st point on } x\text{-axis})$ $+ P(\text{only 2nd point on } x\text{-axis})$ $+ P(\text{both points on } x\text{-axis})$ $= \frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{3}{9}$ $= \frac{24+24+12}{90}$ $= \frac{2}{3}$		
(2) 5 points lie on a coordinate axis $P(\text{both lie on at least one axis})$ $= \frac{5}{10} \times \frac{4}{9}$ $= \frac{2}{9}$	1A 1M 1A	
	11	